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Newtonian and Post-Newtonian Binary Neutron Star Mergers [†]

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We present two of our efforts directed toward the numerical analysis of neutron star mergers, which are the most plausible sources for gravitational wave detectors that should begin operating in the near future. First we present Newtonian 3D simulations including radiation reaction (2.5PN) effects. We discuss the gravitational wave signals and luminosity from the merger with/without radiation reaction effects. Second we present the matching problem between post-Newtonian formulations and general relativity in numerical treatments. We prepare a spherical, static neutron star in a post-Newtonian matched spacetime, and find that discontinuities at the matching surface become smoothed out during fully relativistic evolution if we use a proper slicing condition.

1 Introduction

Mergers of neutron stars in binary systems are considered excellent astrophysical laboratories for gravitational wave astronomy, nuclear astrophysics and relativistic astrophysics. The gravitational waves emitted from these mergers are expected to be observed by gravitational wave detectors coming on-line in the next decade, such as LIGO, VIRGO, GEO and TAMA. Detailed predictions of waveforms are desired and need to be produced analytically and numerically for the extraction of physical information.

Post-Newtonian (PN) calculations [1] can accurately describe the evolution of the system when the separation between the two stars in the binary system is much larger than the stellar radii. The final phase of the neutron star mergers (NSM) requires fully general relativistic (GR) numerical simulations. In this paper, we present two of our efforts at building a bridge between these two approaches.

2 Newtonian simulation of NSM including radiation reaction

First we present Newtonian 3-dimensional simulations including radiation reaction (2.5PN) effects [2]. We note that the Kyoto [3] and Max-Planck [4] groups have also presented results of their NSM simulations using Newtonian equations plus 2.5PN corrections. We use the same set of equations, which are a reduction of the PN-hydro formulation of Blanchet, Damour and Schäfer [5]. We are studying various initial conditions and comparing the effects of different equations of state.

We evolve the Euler equations using a modification of the ZEUS-2D algorithm, staggered grid structure, second order van Leer monotonic interpolator, and Norman's consistent advection method [6]. We have conducted convergence studies to

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delineate how spatial and temporal resolution affect the conservation of angular momentum and energy in these models. In order to get sufficient angular momentum conservation, we find that we must update the "advection" terms before solving the Poisson equations. To solve the Poisson equations, we use a full multi-grid W-cycle algorithm.

Here, we show an example of the gravitational waveform emitted by the coalescence of two equal mass stars. The equation of state is polytropic $P = K\rho^2$ with initial mass $1.4M_\odot$ and radii $R_* = 9.56$Km. The stars are separeted initially by $2.9R_*$, are nonrotating, and have an orbital velocity taken to be the Kepler velocity with infalling radial velocity determined by radiation reaction This simulation was made with 129^3 grid zones ($\Delta x = 0.893$Km) and required 160 hrs of CPU time by Origin 2000 with an average time step 1.43×10^{-3}ms for a total physical time of 5.40 ms.

In Fig.1, we show the gravitational waveform $h_\times$ and luminosity with and without radiation reaction term. We see that the merger occurs earlier when we include radiation reaction terms. This may be explained by the loss of angular momentum from the system induced by radiation reaction. Detail reports are now in preparation [2].

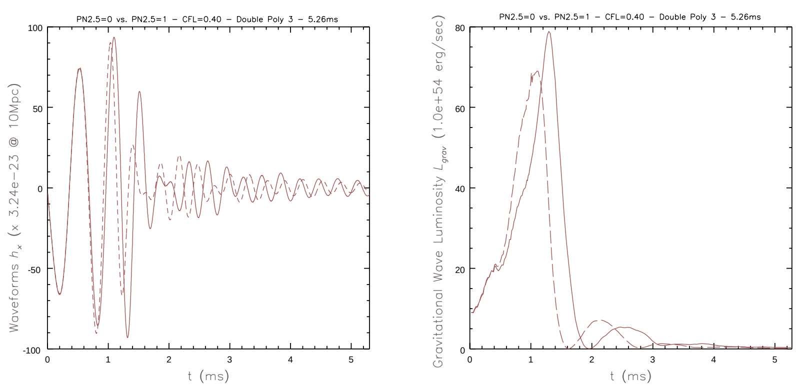

Fig. 1: Waveform $h_\times$ and luminocity. Solid line is Newtonian and dashed line is with radiation reaction.

3 GR evolution of a neutron star with PN matched initial data

Our second effort in linking PN calculations to full GR numerical simulations involves construction of PN initial data for fully relativistic numerical treatment.

As a first step, we constructed a single spherically symmetric neutron star by solving the Tolman-Oppenheimer-Volkoff equations; this solution is matched to a 1PN vacuum metric at some surface outside the star. We then evolved this initial data using a fully relativistic spherical code, and investigated what happens at the matching surface.

In the central region of the star, we set our fully relativistic metric to

$$ds^2 = -e^{2\Phi(R)}dt^2 + e^{2\Lambda(R)}dr^2 + R^2(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right), \qquad (1)$$

where $r = r(R)$. Under the harmonic gauge condition $\frac{d}{dr}\left[e^{\Phi-\Lambda}\rho^2\right] = 2re^{\Phi+\Lambda}$, we solve the hydrostatic equilibrium (TOV) equation and field equations assuming a polytropic equation of state. We define a "mass" M, given by $M = 4\pi\int_0^{R_*}\rho(R)R^2 dR$, where R_* is the radius of the star. We set the exterior metric of the neutron star by $\Phi(R) = \frac{1}{2}ln(1-\frac{2M}{R}) + C$, and $e^{2\Lambda} = (1-\frac{2M}{R})^{-1}$, where $R = r + M$ and the constant term C is to be fixed by matching later. The outer metric (PN region; $r \geq a \geq R_*$, where a is the matching radius) is given by

$$\begin{aligned} ds^2 &= -(1-2U+2U^2+\beta U^3)dt^2 + [1+2U+2U^2+\frac{2}{3}\alpha U^3]dr^2 \\ &\quad +[1+2U+U^2-\frac{1}{3}\alpha U^3]r^2(d\theta^2+\sin\theta d\phi^2)\,, \end{aligned} \tag{2}$$

where $U \equiv \tilde{m}/r$, with $\tilde{m}$ the Kepler-measured mass of the star, β the coefficient of a 2PN term, and α denoting a residual gauge freedom within harmonic gauge. Matching inner and outer metrics at $r = a$ in the standard way yields

$$\tilde{m} = M[1+(M/a)^2+4\,(M/a)^4+O((M/a)^5)]\,, \tag{3a}$$

$$\alpha = 6[1+(M/a)+(M/a)^2+2\,(M/a)^3+O((M/a)^4)]\,, \tag{3b}$$

$$\beta = -\frac{4}{3}[1+(M/a)-2\,(M/a)^2+O((M/a)^3)]\,, \tag{3c}$$

$$C = -\frac{2}{3}\,(M/a)^3-\frac{2}{3}\,(M/a)^4-\frac{37}{15}\,(M/a)^5+O((M/a)^6)]\,. \tag{3d}$$

This initial data contains a discontinuity in the second derivative of the metric at the matching surface $r = a$. After fully relativistic evolution, we find this matching discontinuity to be smoothed out if we use the maximal slicing condition or "K-driver" slicing condition [7], both of which are given by solving an elliptic equation for the lapse function. "K-driver" slicing smoothed out the matching discontinuities faster than the maximal slicing case. Other slicing conditions such as static lapse (harmonic gauge) and the algebraic slicing condition $\alpha = 1 + \log\gamma$ are shown to lead to high frequency noise caused by the discontinuity at the matching surface.

This study suggests that the matching surface is not necessarily fatal, provided suitable slicing conditions are used. Such matching surfaces are expected to exist in constructing initial data using the PN formulation for full GR evolution. Results from simulations using such a PN initial data will be reported in the near future.

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